Tutorial 4: Further Questions (not marked)

1) It is possible for superfluid ^{4}He to offer no resistance when a body moves through it.

i) State the excitations in superfluid ^{4}He.

ii) Before excitation is possible, the body's velocity must be above E/p, where E is energy and p is momentum. State the conservation laws that lead to this.

iii) With the help of the dispersion graph, explain how you could find the critical velocity.

iv) When the body is slower than critical velocity, it experiences no resistance. Why?

2)

i) Bose-Einstein condensate is a good candidate for explaining superfluidity. Why?

ii) The chemical potential μ of a boson gas changes as temperature falls to 0 K. Explain how, using the Bose-Einstein distribution graph.

iii) Write down the formula for the number of bosons N in terms of density of states g(ε), where ε is particle energy. Under certain conditions, the number of excited bosons may be written as

$$N_{ex} = \int_0^\infty \frac{g(\varepsilon)d\varepsilon}{\exp(\varepsilon / k_B T) - 1},$$

where T is temperature. Explain when and why.

iv) The result of the integral in (iii) is

$$N_{ex} = \left(\frac{2\pi m k_B T}{h^2} \right)^{3/2} 2.612V,$$

where V is the volume that contains the N particles. Derive the formula for condensation temperature, T_{BE}. Find T_{BE} for liquid ^{4}He (molar volume 27.58 cm^3).

v) Write down an integral expression for the energy U below T_{BE}, in terms of g(ε). The result of this integral is

$$U = 0.7704 k_B N \frac{T^{5/2}}{T_{BE}^{3/2}} .$$

Derive the heat capacity, C, and find for one mole of ^{4}He at T_{BE}. Sketch the graph of C versus T.